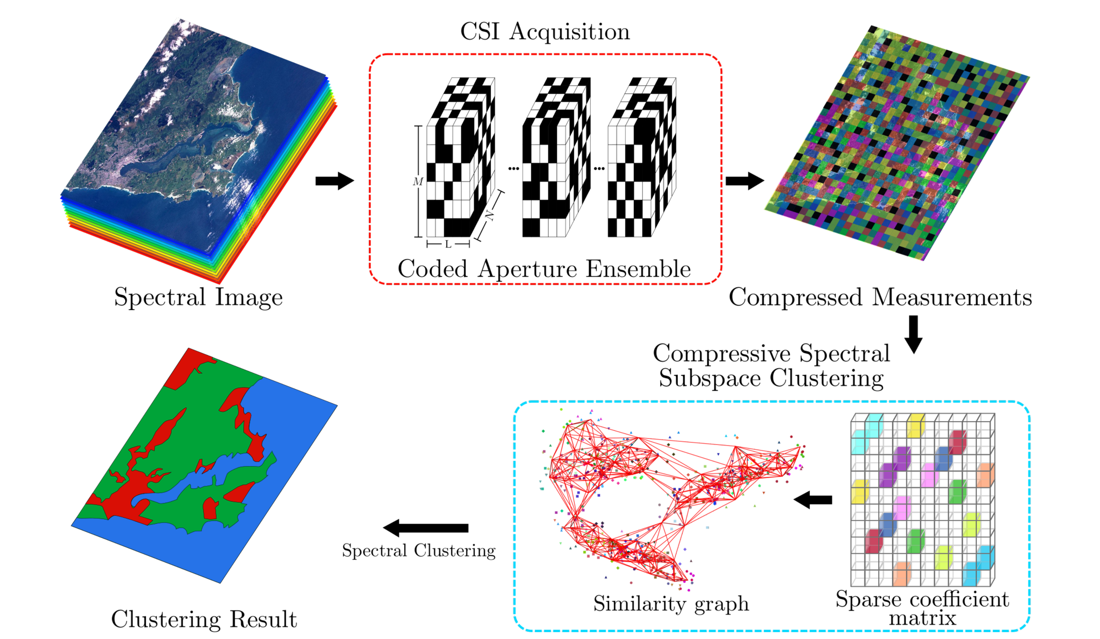
CODED APERTURE DESIGN FOR COMPRESSIVE SPECTRAL SUBSPACE CLUSTERING

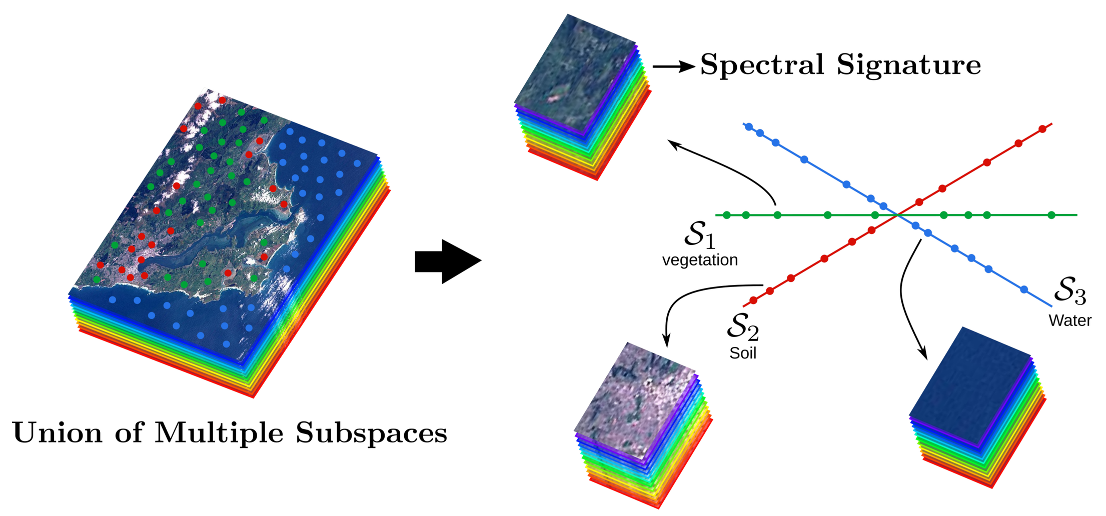
CARLOS HINOJOSA, JORGE BACCA AND HENRY ARGUELLO

{ CARLOS.HINOJOSA@SABER. | JORGE.BACCA1@CORREO. | HENARFU@ } UIS.EDU.CO

WORKFLOW



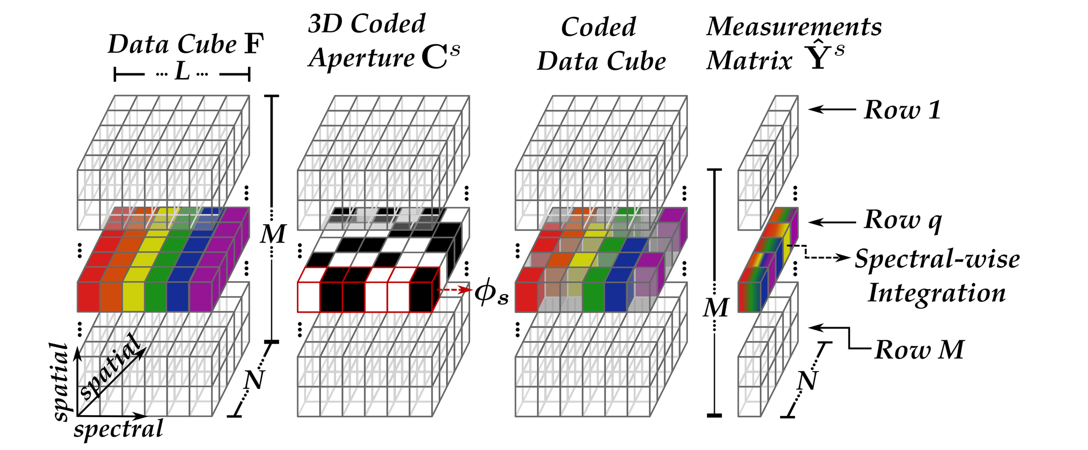
SPARSE SUBSPACE CLUSTERING



- Spectral data points in a union of subspaces are self-expressive, i.e., $\mathbf{Y} = \mathbf{Y}\mathbf{Z}$
- Union of subspaces admits subspace-sparse representation [2]

$$\min_{\mathbf{Z}} \|\mathbf{Z}\|_1 \quad \text{s.t.} \quad \mathbf{Y} = \mathbf{Y}\mathbf{Z} \tag{1}$$

COMPRESSIVE SPECTRAL IMAGING



The acquisition of the compressed measurements can be expressed as

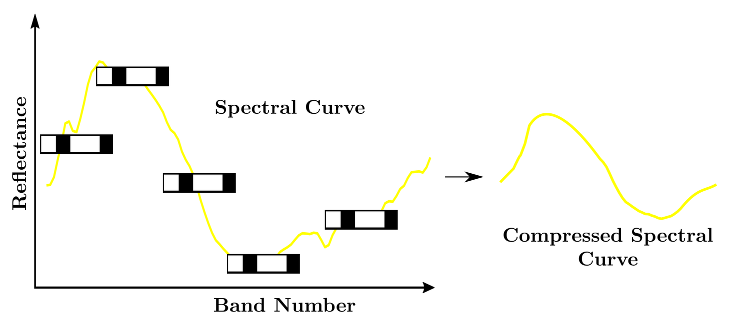
$$\mathbf{Y} = \mathbf{\Phi}\mathbf{F}, \tag{2}$$

where $\mathbf{\Phi} = \left[\phi_1^T, \cdots, \phi_S^T\right]^T$ and $\phi_s \in \mathbb{R}^L$.

CODED APERTURE DESIGN

In order to design the coding patterns matrix $\mathbf{\Phi}$, the following three design criteria are considered

1. Sensing Neighboring Spectral Bands



- Performing the random sampling of neighboring spectral bands will better preserve the information.
- For each coding pattern ϕ^s, select two cutoff wavelengths $(\lambda_1^s, \lambda_2^s) \in \{0, 1, \cdots, L-1\}$ at random such that $\lambda_1^s < \lambda_2^s$ and $\lambda_2^s - \lambda_1^s + 1 = \Delta$. Then

$$(\phi^s)_k = \delta_{\lfloor \lambda_1^2/k \rfloor}\delta_{\lfloor k/\lambda_2^s \rfloor}\varphi_k^s, \tag{3}$$

where $\varphi^s \in \{0,1\}^L$, and δ_x is the Kronecker delta function.

2. Preserving Similarities

- Assuming that the vectors has unit length, the similarity between two compressed measurements $\mathbf{y}_j = \mathbf{\Phi}\boldsymbol{f}_j$, $\mathbf{y}_{j'} = \mathbf{\Phi}\boldsymbol{f}_{j'}$, is defined as

$$\begin{aligned}\text{similarity}(\hat{\mathbf{y}}_j, \hat{\mathbf{y}}_{j'}) &= \hat{\mathbf{y}}_j^T\hat{\mathbf{y}}_{j'} \\ &= \hat{\mathbf{f}}_j^T\mathbf{\Phi}^T\mathbf{\Phi}\hat{\mathbf{f}}_{j'} \quad j \neq j',\end{aligned} \tag{4}$$

- If the columns of $\mathbf{\Phi}$ are normalized, it is possible to decompose the matrix $\mathbf{\Phi}^T\mathbf{\Phi}$ as

$$\mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I} + \boldsymbol{\epsilon}, \tag{5}$$

where

$$\epsilon_{jj'} = \phi_j^T\phi_j' \quad j \neq j', \tag{6}$$

and $\epsilon_{jj} = 0$.

3. Information Acquisition

- In order to better discriminate among the classes, new information from the underlying spectral scene should be acquired in each measurement shot.
- The coding patterns should be linear independent, i.e, the $\mathbf{\Phi}$ matrix should be full rank.
- The number of measurements acquired from each spectral band should be approximately the same.

$$\mathbf{\Phi}\mathbf{\Phi}^T = \mathbf{I} + \boldsymbol{\mu}, \tag{7}$$

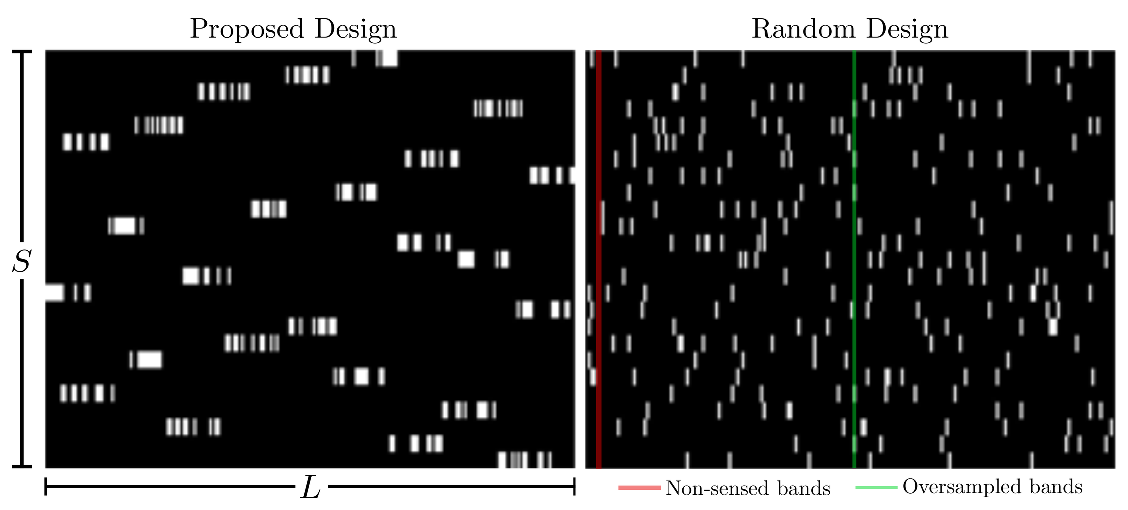
where

$$\mu_{ij} = \phi\phi^T \quad i \neq j, \tag{8}$$

and $\mu_{ii} = 0$.

Optimization Problem

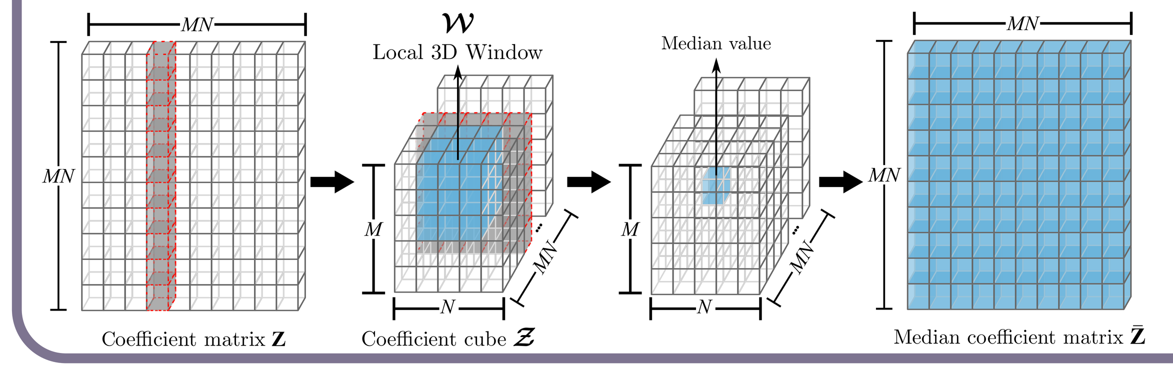
$$\begin{aligned}\underset{\{\mathbf{\Phi},\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\varphi}^s\}}{\arg\min} \quad & f(\mathbf{\Phi}) = \|\mathbf{\Phi}^{\mathbf{T}}\mathbf{\Phi} - \mathbf{I}\|_F^2 + \|\mathbf{\Phi}\mathbf{\Phi}^{\mathbf{T}} - \mathbf{I}\|_F^2 \\ \text{subject to} \quad & (\phi^s)_k = \delta_{\lfloor \lambda_1^s/k \rfloor}\delta_{\lfloor k/\lambda_2^s \rfloor}\varphi_k^s, \\ & \lambda_2^s = \lambda_1^s + \Delta - 1, \\ & \text{Rank}(\mathbf{\Phi}) = S,\end{aligned} \tag{9}$$



COMPRESSIVE SPECTRAL SUBSPACE CLUSTERING

Given $\mathbf{\Phi}$ and $\mathbf{Y}$, the proposed SSC which incorporates spatial information is formulated as follows [3]

$$\begin{aligned}\min_{\mathbf{Z},\mathbf{R},\bar{\mathbf{Z}}} \quad & \|\mathbf{Z}\|_1 + \frac{\lambda}{2}\|\mathbf{R}\|_F^2 + \frac{\alpha}{2}\|\mathbf{Z} - \bar{\mathbf{Z}}\|_F^2 \\ \text{s.t.} \quad & \mathbf{Y} = \mathbf{Y}\mathbf{Z} + \mathbf{R},\ \text{diag}(\mathbf{Z}) = 0,\ \mathbf{Z}^T\mathbf{1} = \mathbf{1},\end{aligned} \tag{10}$$



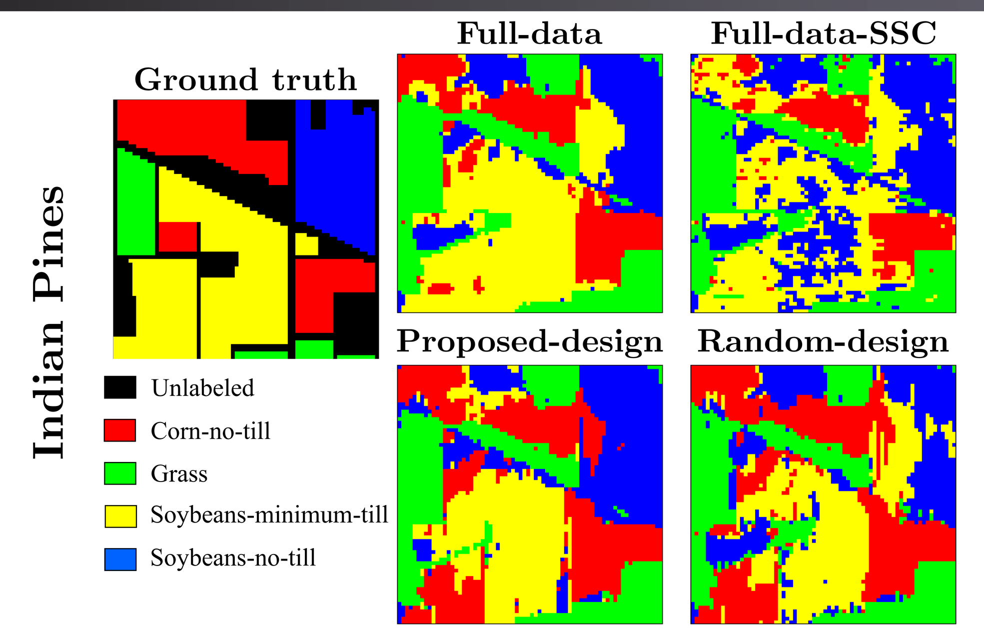
Algorithm Compressive Spectral Subspace Clustering

Input: A set of CSI measurements acquired as $\mathbf{Y} = \mathbf{\Phi}\mathbf{F}$, where the coding pattern matrix $\mathbf{\Phi}$ is obtained by solving optimization problem in (9).

1: Solve the sparse optimization problem in (10).
2: Normalize the columns of $\mathbf{Z}$ as $\mathbf{z}_j \leftarrow \frac{\mathbf{z}_j}{\|\mathbf{z}_j\|_\infty}$
3: Form a similarity graph representing the data points. Set the weights on the edges between the nodes as $\mathbf{W} = |\mathbf{Z}| + |\mathbf{Z}|^T$.
4: Apply SC [1] to the similarity graph.

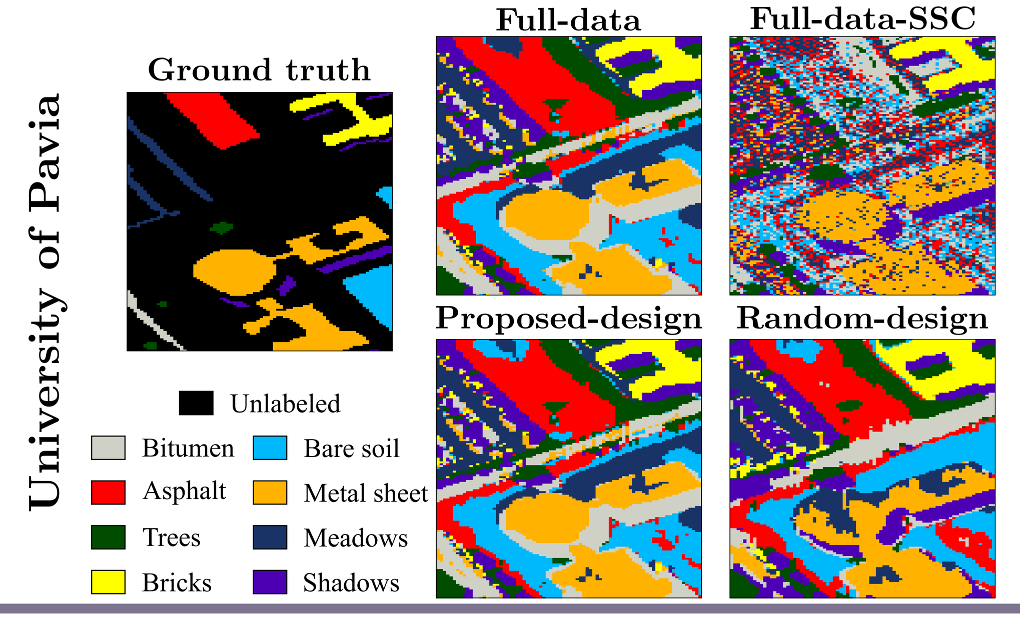
Output: Segmentation of the data: $\boldsymbol{Y}_1, \cdots, \boldsymbol{Y}_\ell$

VISUAL AND QUANTITATIVE RESULTS



QUANTITATIVE EVALUATION OF THE DIFFERENT CLUSTERING RESULTS FOR THE AVIRIS INDIAN PINES IMAGE.

Class	Random-design	Proposed-design	Full-data-SSC	Full-data
Corn-no-till	**73.13**	70.45	48.96	66.77
Grass	95.25	**100**	98.60	**100**
Soybeans-no-till	52.87	**88.80**	70.63	69.54
Soybeans-minimun-till	55.29	60.52	59.23	**80.05**
OA	63.83	73.07	62.62	**76.16**
AA	69.14	**79.94**	69.35	79.09
Kappa	49.26	62.65	47.58	**65.89**

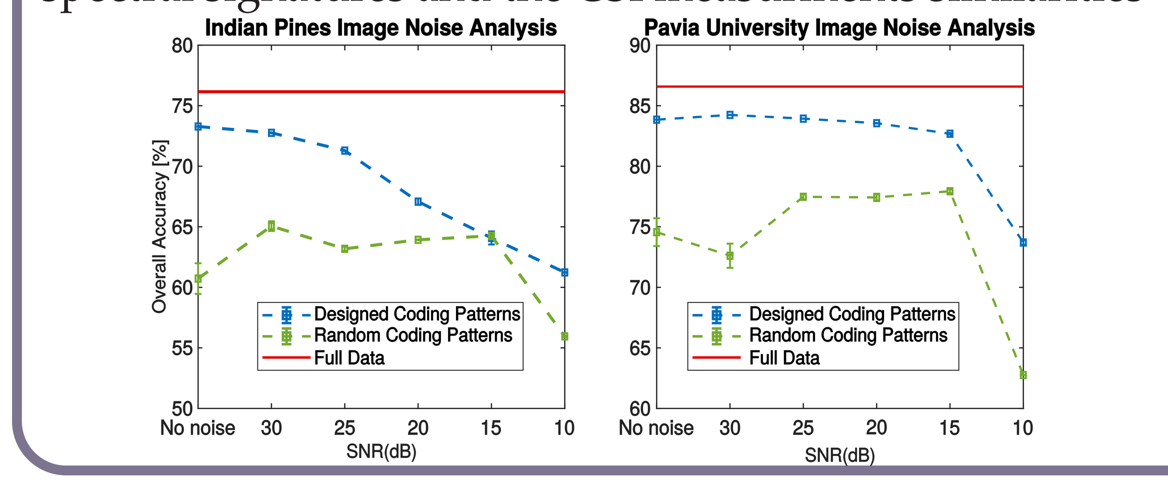


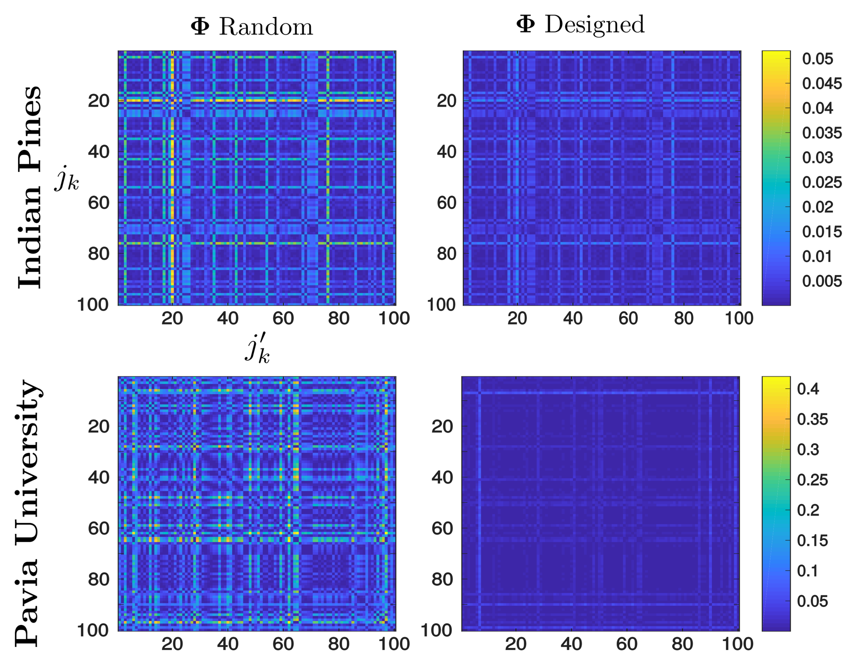
QUANTITATIVE EVALUATION OF THE DIFFERENT CLUSTERING RESULTS WITH THE AVIRIS PAVIA UNIVERSITY IMAGE.

Class	Random-design	Proposed-design	Full-data-SSC	Full-data
Bitumen	18.60	88.37	0	**90.70**
Asphalt	71.37	67.25	33.84	**80.26**
Trees	90.38	88.46	**100**	90.38
Bricks	**100**	99.68	99.68	99.68
Bare Soil	46.78	61.40	36.26	**66.67**
Metal sheet	82.90	**97.73**	91.00	**97.73**
Meadows	91.16	**100**	55.02	**100**
Shadows	**99.48**	24.35	98.45	24.35
OA	78.72	83.81	71.45	**86.58**
AA	75.09	78.41	64.28	**81.22**
Kappa	72.63	78.89	62.95	**82.50**

NOISE AND SIMILARITY PRESERVATION ANALYSIS

The figure below presents the overall clustering accuracy as a function of the aggregated noise. The right hand side figure presents the absolute error between the spectral signatures and the CSI measurments similarities





REFERENCES

[1] U. Von Luxburg A Tutorial on Spectral Clustering. *Statistics and computing*, 17(4), 395-416.

[2] E. Elhamifar and R. Vidal. Sparse Subspace Clustering: Algorithm, Theory, and Applications. *IEEE transactions on pattern analysis and machine intelligence*, 35(11), 2765-2781.

[3] C. Hinojosa, J. Bacca and H. Arguello. Spectral Imaging Subspace Clustering with 3D Spatial Regularizer. *Digital Holography and Three-Dimensional Imaging*(pp. JW5E-7). Optical Society of America.

SOURCE CODE

A GitHub repository with the MatLab codes of this paper can be downloaded from this QR code